Functional Synthesis: An Ideal Meeting Ground for Formal Methods and Machine Learning

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Joint work with: Priyanka Golia [1,2] and Subhajit Roy [2]

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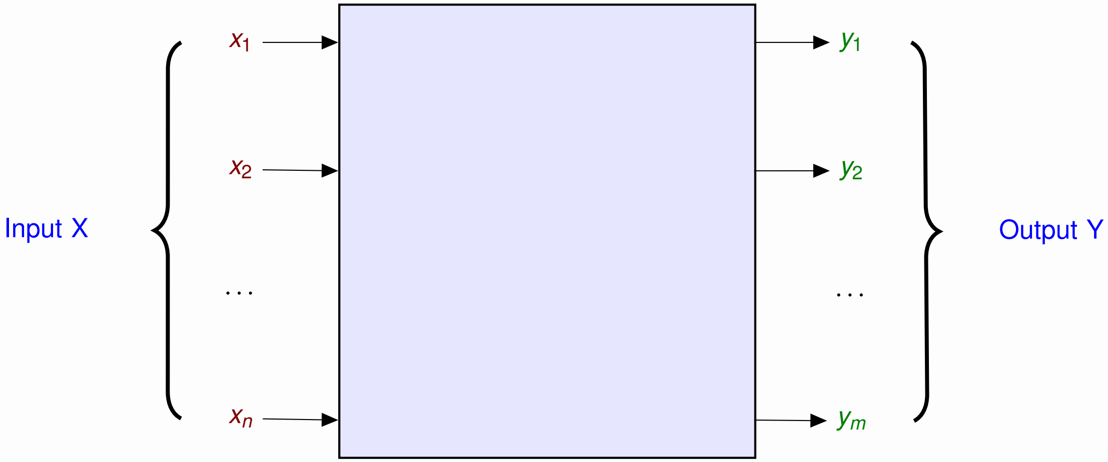
Corresponding Papers: CAV 2020 and ICCAD 2021

Synthesis

Specification: Relation $\varphi(X, Y)$



Synthesis

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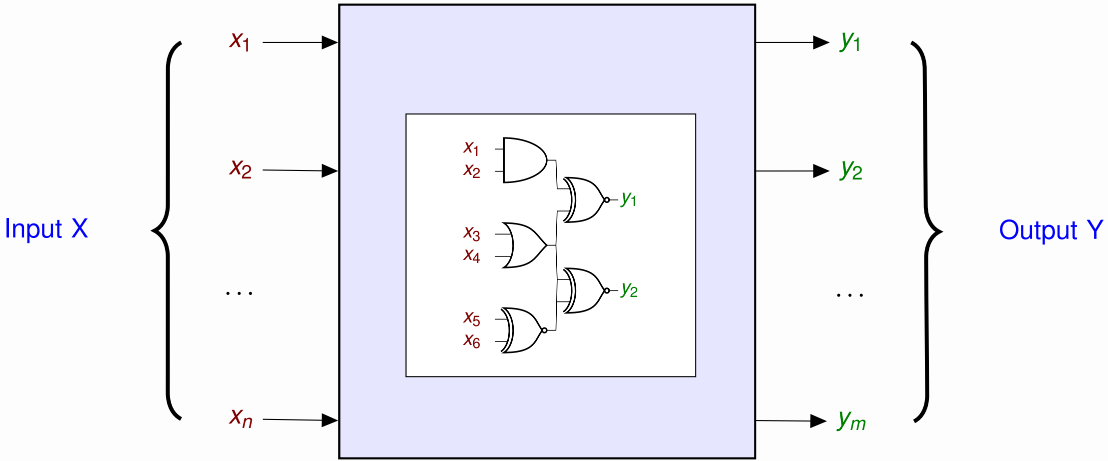
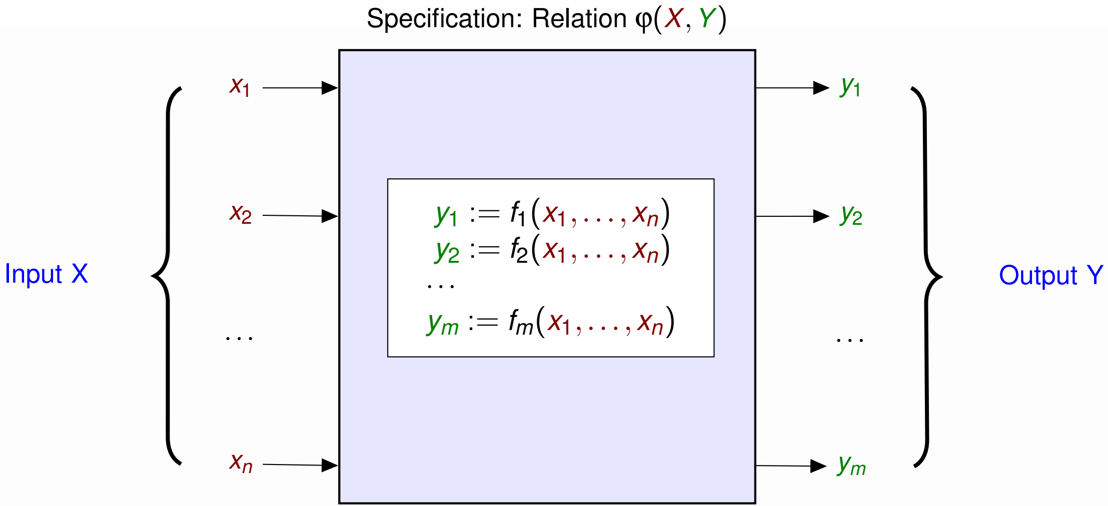
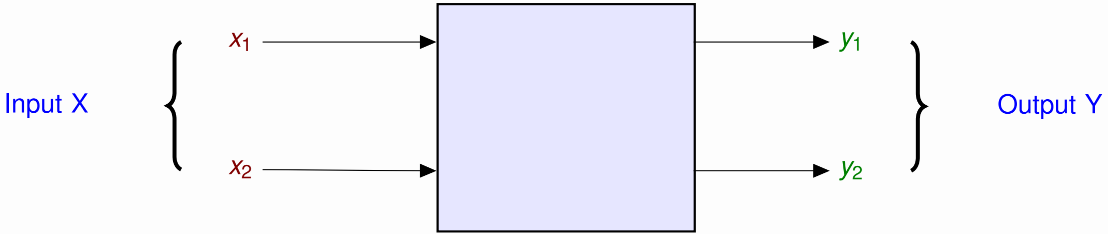


Synthesis


Specification: Relation $\varphi(X, Y)$
Input X
x_1
x_2
$\dots$
x_n
$y_1 := f_1(x_1, \dots, x_n)$
$y_2 := f_2(x_1, \dots, x_n)$
$\dots$
$y_m := f_m(x_1, \dots, x_n)$
y_1
y_2
$\dots$
y_m
Output Y

Synthesis – Example

$$\varphi(X, Y) = (y_1 \geq x_1) \wedge (y_1 \geq x_2) \wedge ((y_1 = x_1) \vee (y_1 = x_2))$$
$$\wedge (y_2 \leq x_1) \wedge (y_2 \leq x_2) \wedge ((y_2 = x_1) \vee (y_2 = x_2))$$



$$\varphi(X, Y) = (y_1 \geq x_1) \wedge (y_1 \geq x_2) \wedge ((y_1 = x_1) \vee (y_1 = x_2))$$
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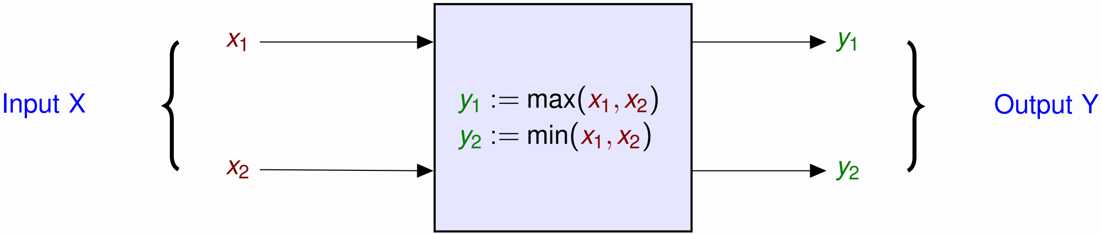

Functional Synthesis

Given $\varphi(X, Y)$ over inputs $X = \{x_1, x_2, \ldots, x_n\}$ and outputs $Y = \{y_1, y_2, \ldots, y_m\}$.

Synthesize A function vector $F = \{f_1, f_2, \ldots, f_m\}$, such that $y_i := f_i(x_1, \ldots, x_n)$ such that:

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

Each f_i is called Skolem function and F is called Skolem function vector.

Key Challenge: $\varphi(X, Y)$ is a relation

Non-uniqueness of Skolem Functions

Let $X = \{x_1, x_2\}$, $Y = \{y_1\}$ and $\varphi(X, Y) = x_1 \vee x_2 \vee y_1$

Possible Skolem function: $f(x_1, x_2) := \neg(x_1 \vee x_2)$

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Possible Skolem function: $f(x_1, x_2) := \neg(x_1 \vee x_2)$

$$\varphi(X, F(X)) = x_1 \vee x_2 \vee (\neg(x_1 \vee x_2))$$

X	$\exists Y \varphi(X, Y)$		$\varphi(X, F(X))$
$x_1 = 0, x_2 = 0$	$y_1 = 1$	True	True
$x_1 = 0, x_2 = 1$	$y_1 = 1$	True	True
$x_1 = 1, x_2 = 0$	$y_1 = 1$	True	True
$x_1 = 1, x_2 = 1$	$y_1 = 1$	True	True

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

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$x_1 = 1, x_2 = 1$	$y_1 = 1$	True	True

$$\exists Y \varphi(X,Y) \equiv \varphi(X, F(X))$$

Other possible Skolem functions: $f_1(x_1, x_2) = \neg x_1 \quad f_1(x_1, x_2) = \neg x_2 \quad f_1(x_1, x_2) = 1$

Diverse Applications

- Quantifier elimination
- Disjunctive decomposition of symbolic transition relations (Trivedi et al.,2002)
- Combinatorial sketching (Solor-Lezma et al 2006, Srivastava et al. 2013)
- Complete functional synthesis (Kuncak et al. 2010)
- Repair/partial synthesis of circuits (Fujita et al. 2010)

Diverse Approaches

- From the proof of validity of $\forall X \exists Y \varphi(X, Y)$

  (Bendetti et al., 2005)
  (Jussilla et al., 2007)
  (Heule et al., 2014)
- Quantifier instantiation in SMT solvers

  (Barrett et al., 2015)
  (Bierre et al.,2017)
- Input-Output Separation

  (Chakraborty et al., 2018)
- Knowledge representation

  (Kukula et al., 2000)
  (Trivedi et al., 2003)
  (Jiang, 2009)
  (Kuncak et al., 2010)
  (Balabanov and Jiang, 2011)
  (John et al., 2015)
  (Fried, Tabajara, Vardi, 2016,2017)
  (Akshay et al., 2017,2018)
  (Chakraborty et al., 2019)
- Incremental determinization

  (Rabe et al., 2015, 2018, 2019)

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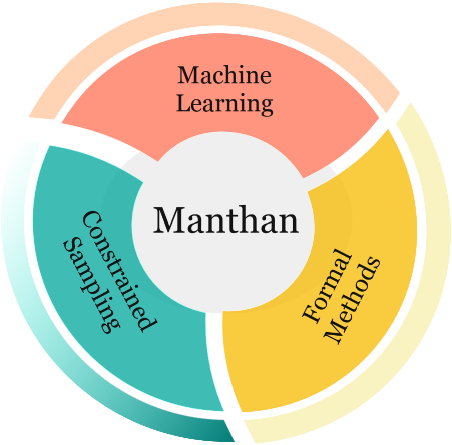
Scalability remains the holy grail

A Data-Driven Approach for Boolean Functional Synthesis



François Chollet
@fchollet

Machine
~~Deep~~ learning excels at unlocking the creation of impressive early demos of new applications using very little development resources.

The part where it struggles is reaching the level of consistent usefulness and reliability required by production usage.

Take Away Message

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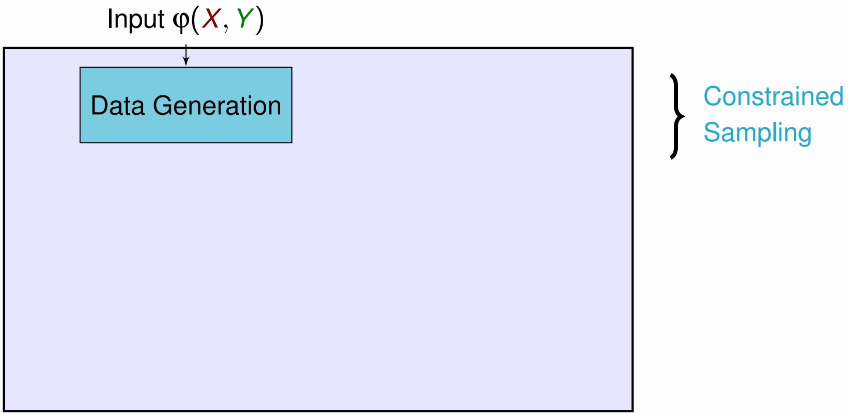
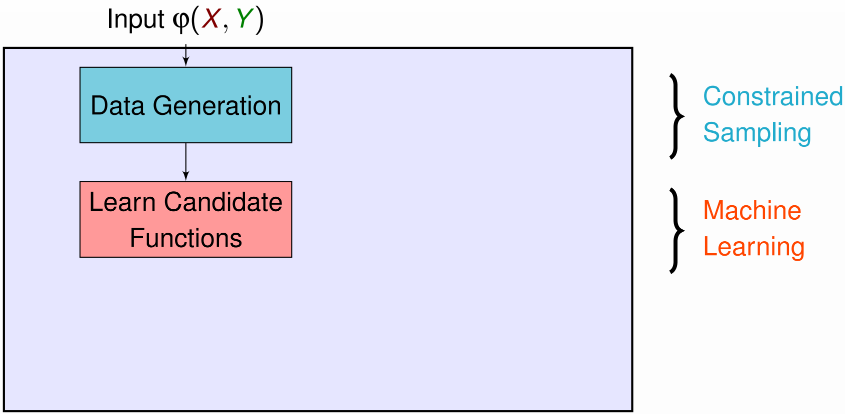
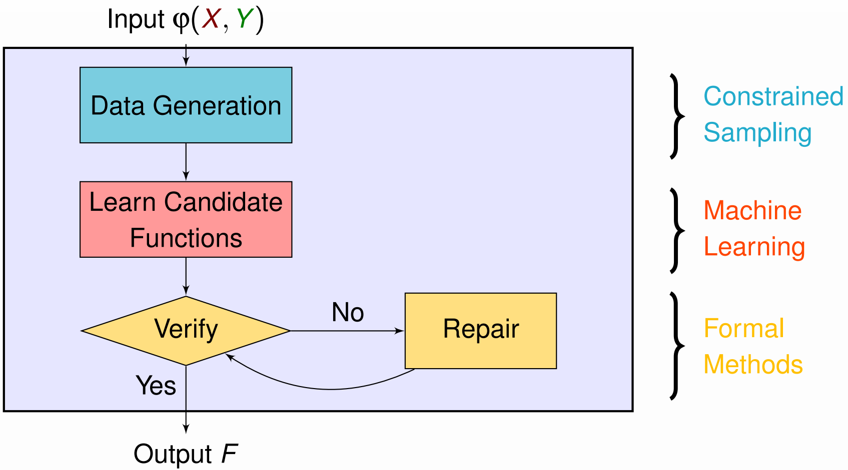
Formal Methods is the Answer to Machine Learning's Struggles



Input φ(X, Y)
Data Generation
Constrained
Sampling

Manthan

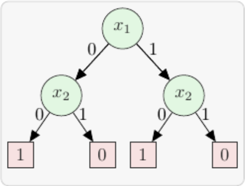

Input φ(X, Y)
Data Generation
Learn Candidate Functions
Constrained Sampling
Machine Learning


Input φ(X, Y)
Data Generation
Learn Candidate Functions
Verify
No
Repair
Yes
Output F
Constrained Sampling
Machine Learning
Formal Methods

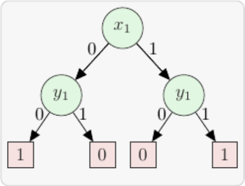
$$\varphi(x_1, x_2, y_1, y_2) \longrightarrow$$

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0



$p_1 := (\neg x_1 \wedge \neg x_2)$,
$p_2 := (x_1 \wedge \neg x_2)$
f_1 = if p_1 then 1
elif p_2 then 1
else 0



$p_1 := (\neg x_1 \wedge \neg y_1)$,
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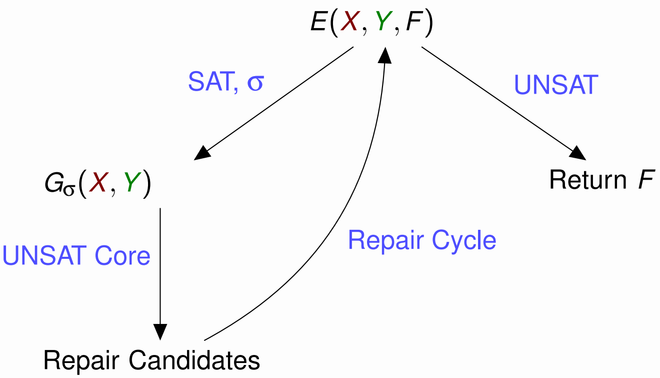
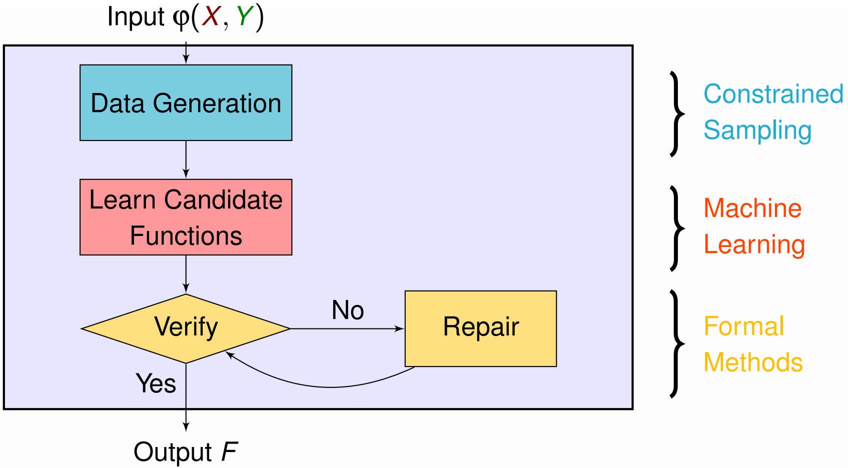
else 0



$E(X, Y, F)$
SAT, σ
UNSAT
$G_\sigma(X, Y)$
Return F
UNSAT Core
Repair Cycle
Repair Candidates


Input $\varphi(X, Y)$
Data Generation
Learn Candidate Functions
Verify
No
Repair
Yes
Output F
Constrained Sampling
Machine Learning
Formal Methods

Data Generation

Potential Strategy: Randomly sample satisfying assignment of $\varphi(X, Y)$.

Challenge: Multiple valuations of y_1, y_2 for same valuation of x_1, x_2.

Data Generation

Potential Strategy: Randomly sample satisfying assignment of $\varphi(X, Y)$.

Challenge: Multiple valuations of y_1, y_2 for same valuation of x_1, x_2.

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

$\xrightarrow{\text{Uniform Sampler}}$

x_1	x_2	y_1	y_2
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

$\xrightarrow{\text{Uniform Sampler}}$

x_1	x_2	y_1	y_2
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$

Data Generation

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

x_1	x_2	y_1	y_2
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0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

$\xrightarrow{\text{Uniform Sampler}}$

x_1	x_2	y_1	y_2
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad f_1(x_1, x_2) = \neg x_1$ $\quad f_1(x_1, x_2) = \neg x_2$ $\quad f_1(x_1, x_2) = 1$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$ $\quad f_2(x_1, x_2) = \neg x_1$ $\quad f_2(x_1, x_2) = \neg x_2$ $\quad f_2(x_1, x_2) = 0$

Data Generation

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

x_1	x_2	y_1	y_2
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

$\xrightarrow{\text{Magical Sampler}}$

x_1	x_2	y_1	y_2
0	0	1	0
0	1	1	0
1	0	1	0
1	1	1	0

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad f_1(x_1, x_2) = \neg x_1$ $\quad f_1(x_1, x_2) = \neg x_2$ $\quad f_1(x_1, x_2) = 1$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$ $\quad f_2(x_1, x_2) = \neg x_1$ $\quad f_2(x_1, x_2) = \neg x_2$ $\quad f_2(x_1, x_2) = 0$

Weighted Sampling to Rescue

- $W : X \cup Y \mapsto [0, 1]$

- The probability of outputting an assignment is proportional to its weight.

$$W(\sigma) = \prod_{\sigma(z_i)=1} W(z_i) \prod_{\sigma(z_i)=0} (1 - W(z_i))$$

- Example: $W(x_1) = 0.5 \quad W(x_2) = 0.5 \quad W(y_1) = 0.9 \quad W(y_2) = 0.1$
  $\sigma_1 = \{x_1 \mapsto 1, x_2 \mapsto 0, y_1 \mapsto 0, y_2 \mapsto 1\}$

$$W(\sigma_1) = 0.5 \times (1 - 0.5) \times (1 - 0.9) \times 0.1 = 0.0025$$

- Uniform sampling is a special case where all variables are assigned weight of 0.5.

Data Generation

Generate Samples with
$W(x_i) = 0.5$
$W(y_i) = 0.9$

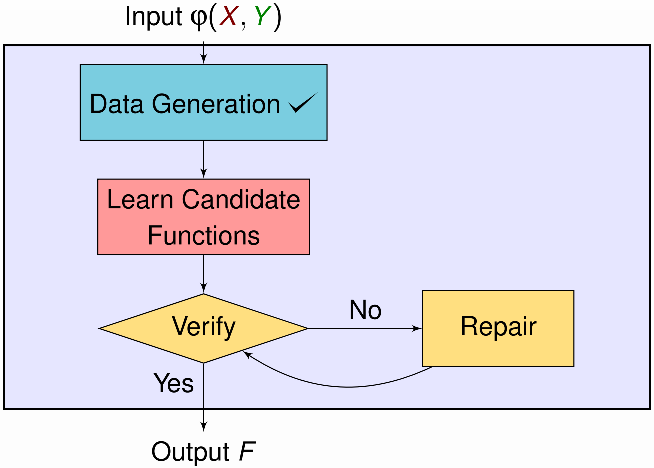
Generate Samples with
$W(x_i) = 0.5$
$W(y_i) = 0.1$

Compute Weights q_i

Generate Samples with
$W(x_i) = 0.5$
$W(y_i) = q_i$

Different Sampling Strategies

- Knowledge representation based techniques
  (Yuan,Shultz, Pixley,Miller,Aziz 1999)
  (Yuan,Aziz, Pixley,Albin, 2004)
  (Kukula and Shiple, 2000)
  (Sharma, Gupta, M., Roy, 2018)
  (Gupta, Sharma, M., Roy, 2019)
- Hashing based techniques
  (Chakraborty, M., and Vardi 2013, 2014,2015)
  (Soos, M., and Gocht 2020)
- Mutation based techniques
  (Dutra, Laeufer, Bachrach, Sen, 2018)
- Markov Chain Monte Carlo based techniques
  (Wei and Selman,2005)
  (Kitchen,2010)
- Constraint solver based techniques
  (Ermon, Gomes, Sabharwal, Selman,2012)
- Belief networks based techniques
  (Dechter, Kask, Bin, Emek,2002)
  (Gogate and Dechter,2006)


Input φ(X, Y)
Data Generation ✓
Learn Candidate Functions
Verify
No
Repair
Yes
Output F

Learn Candidate Function: Decision Tree Classifier

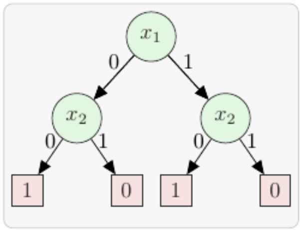
$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$$

- To learn y_2
  - Feature set: valuation of x_1, x_2, y_1
  - Label: valuation of y_2
  - Learn decision tree to represent y_2 in terms of x_1, x_2, y_1
- To learn y_1
  - Feature set: valuation of x_1, x_2
  - Label: valuation of y_1
  - Learn decision tree to represent y_1 in terms of x_1, x_2

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0

Learning Candidate Functions

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0



$$
\begin{aligned}
p_1 &:= (\neg x_1 \wedge \neg x_2), \\
p_2 &:= (x_1 \wedge \neg x_2) \\
f_1 &= \text{if } p_1 \text{ then } 1 \\
&\quad\ \text{elif } p_2 \text{ then } 1 \\
&\quad\ \text{else } 0
\end{aligned}
$$

Learning Candidate Functions

x_1	x_2	y_1	y_2
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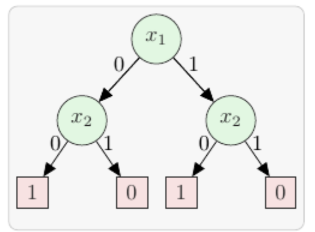

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What Kind of Learning

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
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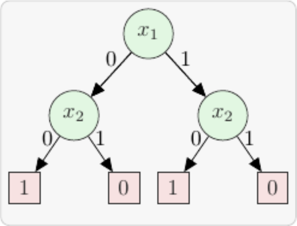

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Learning without Error
Every row is a solution of $\varphi(X, Y)$

Learning with Errors
The data is only a subset of solutions.

What Kind of Learning

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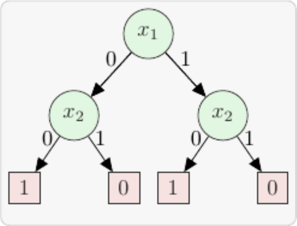

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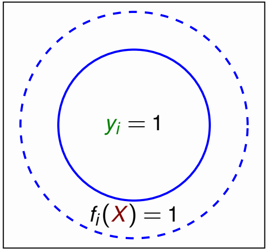
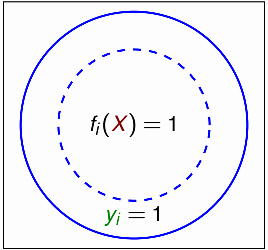
Learning without Error
Every row is a solution of $\varphi(X, Y)$

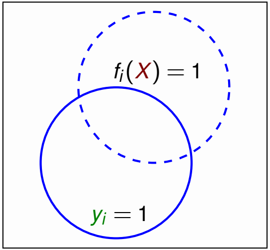
Learning with Errors
The data is only a subset of solutions.

Learn with Errors: Approximations <u>not</u> Abstractions

Abstraction vs Approximation







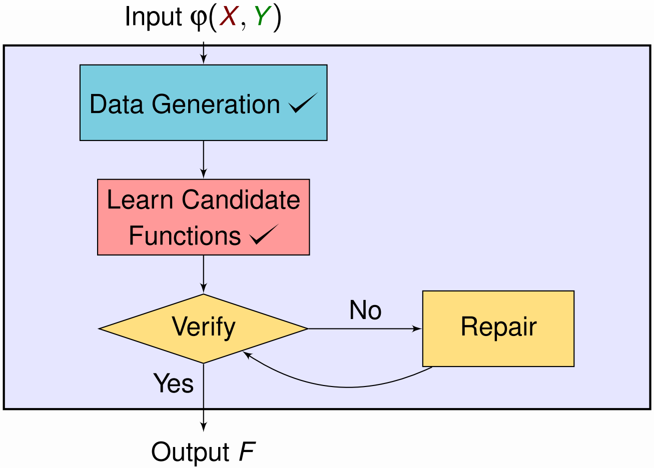
$y_i \rightarrow f_i(X)$ $\qquad$ $f_i(X) \rightarrow y_i$

Abstraction

Approximation

$y_i = 1, f_i(X) = 0$

$y_i = 0, f_i(X) = 1$


Input φ(X, Y)
Data Generation ✓
Learn Candidate Functions ✓
Verify
No
Repair
Yes
Output F

Verification of Candidate Functions

$$E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$$

(JSCTA'15)

- If $E(X, Y, Y')$ is UNSAT: $\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$
  - Return F
- If $E(X, Y, Y')$ is SAT: $\exists Y \varphi(X, Y) \not\equiv \varphi(X, F(X))$
  - Let $\sigma \models E(X, Y, Y')$ be a counterexample to fix.

Repair Candidate Identification

$$E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$$

$$\sigma \models E(X, Y, Y') \text{ be a counterexample to fix.}$$

- Let $\sigma := \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 1, y_2 \mapsto 1, y'_1 \mapsto 0, y'_2 \mapsto 0\}$.
- Potential repair candidates: All y_i where $\sigma[y_i] \neq \sigma[y'_i]$.

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- $\varphi(X, Y)$ is Boolean Relation.
  - So it can be $\hat{\sigma} = \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 0, y_2 \mapsto 1, y'_1 \mapsto 0, y'_2 \mapsto 0\}$
  - We would not repair f_1.

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  - We would not repair f_1.
- MaxSAT-based Identification of *nice counterexamples*:
  - Hard Clauses $\varphi(X, Y) \wedge (X \leftrightarrow \sigma[X])$.
  - Soft Clauses $(Y \leftrightarrow \sigma[Y'])$.
- Candidates to repair: Y variables in the violated soft clauses

Repairing Approximations

- $\sigma = \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 0, y_2 \mapsto 1, y_1' \mapsto 0, y_2' \mapsto 0\}$, and we want to repair f_2.
- Potential Repair: If $\underbrace{x_1 \land x_2 \land \neg y_1}_{\beta=\{x_1, x_2, \neg y_1\}}$ then $y_2 = 1$
- Would be nice to have $\beta = \{x_1, x_2\}$ or even $\beta = \{x_1\}$
- Challenge: How do we find small β?
  - $G_\sigma(X, Y) := \varphi(X, Y) \land x_1 \land x_2 \land \neg y_1 \land (y_2 = 0)$
  - β:= Literals in UNSAT Core of $G_\sigma(X, Y)$

$\varphi(X, Y)$

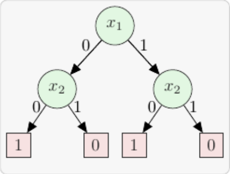
$X = \{x_1, x_2\}$

$Y = \{y_1, y_2\}$

Data Generation

x_1	x_2	y_1	y_2
0	0	1	0
0	1	0	1
1	0	1	0
1	1	0	1

Learn Candidates



Verify Candidates

Check Satisfiability of $E(X, Y, Y')$

SAT, σ

$G_\sigma(X, Y)$

UNSAT Core-based Repair

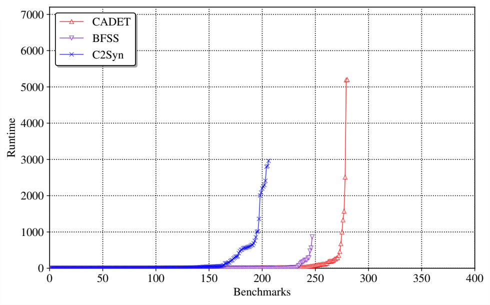
UNSAT

Return F

Experimental Evaluations

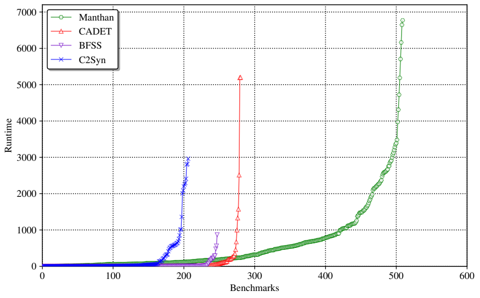
- 609 Benchmarks from:
  - QBFEval competition
  - Arithmetic
  - Disjunctive decomposition
  - Factorization
- Compared Manthan with State-of-the-art tools: CADET (Rabe et al., 2019), BFSS (Akshay et al. ,2018), C2Syn (Chakraborty et al., 2019).
- Timeout: 7200 seconds.

Experimental Evaluations



C2Syn	BFSS	CADET
206	247	280

Experimental Evaluations



C2Syn	BFSS	CADET	Manthan
206	247	280	509

An increase of 223 benchmarks.

Future work: Interesting Questions

- Learning Theoretic Foundations for Functional Synthesis
  - What is the ideal distribution to generate the data?
  - Mistake bounds/complexity of learning functions from relations?
- The Future of Formal Methods (FM) +Machine Learning (ML)
  - The proposed solutions by ML do not need to be fully correct.
  - Use FM for correctness and ML to quickly find the solution.

Conclusion

Manthan: A Data-Driven Approach for Boolean Functional Synthesis.

- Constrained Sampling
- Decision List Classifier
- Formal Methods

Solves 509 benchmarks — state of the art could solve 280

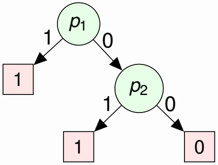
https://github.com/meelgroup/manthan

Thanks!

Repair: Adding Level to Decision List

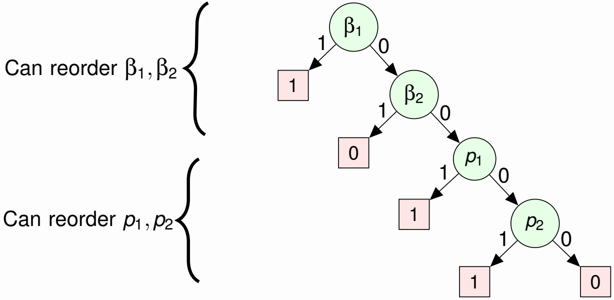
- Candidates are from one level decision list:
  - Say we have paths p_1, p_2 with the leaf node label as 1.
  - Learned decision tree: If p_1 then 1, elif p_2 then 1, else 0.
  - p_1, p_2 can be reordered.

Can reorder p_1, p_2



Repair: Adding Level to Decision List

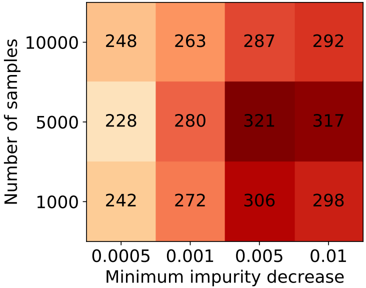
- Candidates are from one level decision list:
  - Say we have paths p_1, p_2 with the leaf node label as 1.
  - Learned decision tree: If p_1 then 1, elif p_2 then 1, else 0.
  - p_1, p_2 can be reordered.
- Suppose in repair iterations, we have learned: If β_1 then 1, $\ldots\beta_2$ then 0 $\ldots\ldots$
- β_1 and β_2 can be reordered.
- From one level decision list to two decision list.



Impact of different sampling schemes and the quality of samplers.

Sampler	Instances Solved with No Repair	Total Instances Solved
CryptoMiniSAT	14	271
QuickSampler	28	275
Uniform Sampler	51	345
Weighted Sampler	66	356

Learning Candidate Functions: Experimental Evaluations(I)



- Learning without any errors on sampled data: Manthan could only solves 162 instances.
- Manthan decides the number of sampler as per cardinality of Y variables, and uses 0.005 as minimum impurity decrease parameter.

Manthan: Example

- Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$
- $\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$
- Skolem Functions:
  - $f_1(x_1, x_2) := (x_1 \vee x_2)$
  - $f_2(x_1, x_2, y_1) := (x_1 \wedge (x_2 \vee y_1))$
    $f_2(x_1, x_2, y_1) := (x_1 \wedge (x_2 \vee (x_1 \vee x_2))$
    $f_2(x_1, x_2, y_1) := x_1$

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

Example: Data Generation

Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

$$\Big\downarrow \text{Constrained Sampler}$$

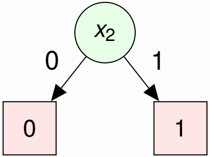
x_1	x_2	y_1	y_2
0	0	0	0
0	1	1	0
1	1	1	1

Example: Learning Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- Learn candidate function f_1.
- Feature set for y_1 := $\{x_1, x_2\}$

x_1	x_2	y_1
0	0	0
0	1	1
1	1	1



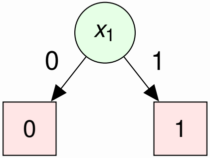
$$f_1(x_1, x_2) := x_2$$

Example: Learning Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- Learn candidate function f_2.
- Feature set for $y_2 := \{x_1, x_2, y_1\}$

x_1	x_2	y_1	y_2
0	0	0	0
0	1	1	0
1	1	1	1



$$f_2(x_1, x_2, y_1) := x_1$$

Example: Verification of Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X, Y, Y') := \varphi(x_1, x_2, y_1, y_2) \wedge \neg\varphi(x_1, x_2, y'_1, y'_2) \wedge (y'_1 \leftrightarrow x_2) \wedge (y'_2 \leftrightarrow x_1)$$

$$\downarrow$$

SAT

$$\downarrow$$

$$\sigma \models E(X, Y, Y') \longrightarrow \sigma[x_1] = 1, \sigma[x_2] = 0$$

$$\swarrow \qquad \searrow$$

$$\sigma[y_1] = 1,\ \sigma[y_2] = 1 \qquad \sigma[y'_1] = 0,\ \sigma[y'_2] = 1$$

Example: Verification of Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X, Y, Y') := \varphi(x_1, x_2, y_1, y_2) \wedge \neg\varphi(x_1, x_2, y'_1, y'_2) \wedge (y'_1 \leftrightarrow x_2) \wedge (y'_2 \leftrightarrow x_1)$$

$$\downarrow$$

SAT

$$\downarrow$$

$$\sigma \models E(X, Y, Y') \longrightarrow \sigma[x_1] = 1, \sigma[x_2] = 0$$

$$\sigma[y_1 = 1], \sigma[y_2] = 1 \qquad \sigma[y'_1 = 0], \sigma[y'_2] = 1$$

$\sigma[y_1] \neq \sigma[y'_1]$

Candidate to repair f_1

Example: Repairing candidate functions (I)

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $G_1(X, Y) = \varphi(X, Y) \wedge (X \leftrightarrow \sigma[X]) \wedge (y_1 \leftrightarrow \sigma[y_1']$.
- $G_1(X, Y) = \varphi(X, Y) \wedge (x_1 \leftrightarrow 1) \wedge (x_2 \leftrightarrow 0) \wedge (y_1 \leftrightarrow 0)$.
- UNSAT core of $G_1(X, Y) = \varphi(X, Y) \wedge (x_1 \leftrightarrow 1) \wedge (y_1 \leftrightarrow 0)$
- Repair formula $\beta = x_1$.

Example: Repairing candidate functions (II)

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

Before repair	Repair	After repair
$f_1(\sigma[X]) \mapsto 0$	$f_1(X) \leftarrow f_1(X) \vee \beta$ $f_1(X) \leftarrow x_2 \vee x_1$	$f_1(X) \mapsto 1$

Example: Verification of Candidate Functions

$$\varphi(X,Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X,Y,Y') := \varphi(X,Y) \wedge \neg\varphi(X,Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X,Y,Y') := \varphi(x_1,x_2,y_1,y_2) \wedge \neg\varphi(x_1,x_2,y_1',y_2') \wedge (y_1' \leftrightarrow x_2 \vee x_1) \wedge (y_2' \leftrightarrow x_1)$$

$$\downarrow$$

UNSAT

$$\downarrow$$

Manthan returns F

Data Generation

- $\Sigma_1 :=$ Sample 500 data point with $W(x_i) = 0.5$ and $W(y_i) = 0.9$.

$$w_1(i) = \frac{\text{Count}(\Sigma_1 \cap (y_i = 1))}{500}$$

- $\Sigma_2 :=$ Sample 500 data point with $W(x_i) = 0.5$ and $W(y_i) = 0.1$.

$$w_2(i) = \frac{\text{Count}(\Sigma_2 \cap (y_i = 0))}{500}$$

- If $0.35 < w_1(i) < 0.65$ and $0.35 < w_2(i) < 0.65$, then $q_i = w_1(i)$, else $q_i = 0.9$.